Weak Qubit Measurement with a Nonlinear Cavity: Beyond Perturbation Theory

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(Received 25 June 2012; published 19 September 2012)

We analyze the use of a driven nonlinear cavity to make a weak continuous measurement of a dispersively coupled qubit. We calculate the backaction dephasing rate and measurement rate beyond leading-order perturbation theory using a phase-space approach which accounts for cavity noise squeezing. Surprisingly, we find that increasing the coupling strength beyond the regime describable by leading-order perturbation theory (i.e., linear response) allows one to come significantly closer to the quantum limit on the measurement efficiency. We interpret this behavior in terms of the non-Gaussian photon number fluctuations of the nonlinear cavity. Our results are relevant to recent experiments using superconducting microwave circuits to study quantum measurement.

DOI: 10.1103/PhysRevLett.109.123602 PACS numbers: 42.50.Dv, 03.65.Yz, 42.50.Pq, 85.25.Cp

Introduction.—There is considerable interest in exploiting continuous weak quantum measurements for the detection of fundamental quantum behavior as well as for quantum information processing [1–3]. By weak measurement, we mean the generic situation where the signal produced in the detector by the measured system is small compared to intrinsic output noise, and thus information is obtained only gradually in time. Such measurements are ultimately constrained by the Heisenberg uncertainty principle, which dictates that the backaction disturbance of the system by the detector cannot be arbitrarily small, but is instead bounded by the rate at which information is acquired [1–5]. Detectors capable of yielding an optimally small ratio of backaction-to-information gain are known as quantum limited. They are both of fundamental interest, and are also necessary if one wishes to implement continuous quantum feedback algorithms [6–8] or certain quantum error correction schemes [9].

Not surprisingly, weak measurements are usually analyzed in the limit of a system-detector coupling small enough that leading-order perturbation theory in the coupling applies; in this standard regime, the quantum limit reduces to a constraint on the noise properties of the detector [3,4]. Here, we focus on an alternate regime, where the detector-system coupling is still weak enough that information is obtained gradually in time, but not so weak that leading-order perturbation theory is sufficient. This regime of a "weak-but-not-too-weak" measurement has recently been achieved in experiments using a driven, nonlinear superconducting microwave cavity to measure the state of a superconducting qubit [10]. As with experiments using linear microwave cavities [11,12], the qubit is dispersively coupled to the cavity, meaning that the cavity frequency depends on the qubit state; by monitoring the phase of reflected microwaves from the cavity, one can monitor the qubit state. Introducing a nonlinearity in the cavity via a Josephson junction (see Fig. 1) allows one to operate the cavity detector close to a point of bifurcation, where the state of the driven cavity is an extremely sensitive (but still single-valued) function of its frequency. The enhanced sensitivity of this regime naturally leads to conditions where the measurement is weak, but the qubit-detector coupling cannot be treated perturbatively. While information gain is enhanced here, the question remains whether this speedup comes at the cost of deviating from the quantum limit (i.e., excess backaction dephasing).

In this Letter, we present an analytic theory describing weak measurement of a qubit with a nonlinear cavity operated close to a point of bifurcation. Our nonperturbative

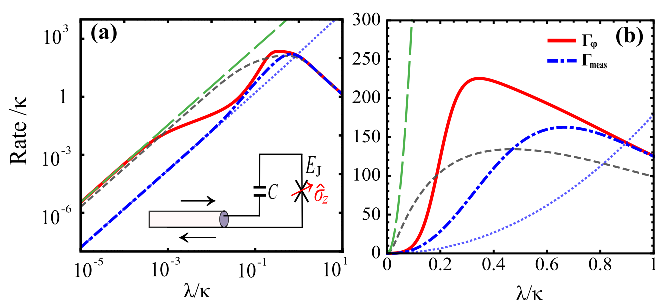

FIG. 1 (color online). (a) Inset: schematic showing one possible realization of a Josephson-junction circuit (i.e., nonlinear cavity) dispersively coupled to a qubit. Main: Measurement rate and dephasing rate versus qubit coupling strength λ, in units of the cavity damping rate κ, using logarithmic axes. The blue (dot-dashed) curve is the full measurement rate Γ_{meas}, while the light-blue (dotted) curve is the linear-response approximation to Γ_{meas}. The red (solid) line is the dephasing rate Γ_φ as obtained from the full theory presented in the main text. The remaining lines are Γ_φ calculated within less rigorous approximations: the grey (short-dashed) curve is the linear-cavity formula $\Gamma_{\varphi,0}$ [cf. Eq. (5)] and green (long-dashed) curve is leading-order perturbation theory. Parameters are $\Lambda = 10^{-3}\kappa$, $f_0 = 0.75 f_{\text{bif}}$, $\bar{n} \sim 200$, $\Delta = \Delta_{\text{bif}}$ where f_{bif} and Δ_{bif} are the driving force amplitude and detuning at the cavity bifurcation. The parametric photon number gain is $G \sim 10^2$. One clearly sees that for moderate couplings, the dephasing rate is strongly suppressed compared to the perturbative result. (b) Same, but using linear axes.

approach accounts for the nontrivial cavity noise physics associated with the nonlinearity. We find that the information-gain to state-disturbance ratio is a strong function of the qubit-detector coupling strength. In the limit of an extremely weak qubit-detector coupling, we recover previous perturbative results [13–16], which indicate a large deviation from the quantum limit: the backaction dephasing rate is a large factor G greater than the rate of information acquisition (the measurement rate), where $G \gg 1$ is the parametric photon-number gain associated with the driven nonlinear cavity (i.e., a single signal photon incident on the detector will result in G photons at its output). Increasing the coupling beyond the perturbative regime, we find remarkably that the dephasing rate is greatly suppressed compared to the leading-order prediction. This allows one to approach the quantum limit to within a factor of order unity. Our approach provides a general framework for investigating quantum measurement with driven nonlinear systems beyond weak coupling.

Note that the backaction of a nonlinear cavity detector was also considered by Boissonneault and co-workers [10,15,16]. They described backaction dephasing beyond lowest-order in the coupling by approximating the state of the driven cavity state *conditioned* on the qubit state to be a simple coherent state; this is only valid for operating points far from a bifurcation, where the detector has a relatively small gain. We show that close to a bifurcation (where the cavity exhibits parametric gain and squeezing), this approach does not accurately capture the coupling dependence of the backaction dephasing.

Model.—We consider a qubit coupled dispersively to a single-sided nonlinear cavity. While our approach applies to an arbitrary nonlinearity, we focus here on the typical experimental situation [10,17] where a Kerr-type nonlinearity dominates. Working in a frame rotating at the cavity drive frequency ω_d, the Hamiltonian is ($\hbar = 1$)

$$\hat{H}_{\rm sys} = -\Delta\hat{a}^\dagger\hat{a} - \Lambda\hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a} + \hat{H}_\kappa + \hat{H}_{\rm qb} + \hat{H}_{\rm int}, \quad (1)$$

where $\Delta = \omega_d - \omega_{\rm cav}$ is the detuning between the cavity drive and resonance frequency, Λ is the Kerr constant, $\hat{H}_\kappa$ describes the cavity damping (with rate κ) and driving due to coupling to external modes, and $\hat{H}_{\rm qb} = \Omega\hat{\sigma}_z$ is the qubit Hamiltonian. The dispersive quantum nondemolition qubit-cavity coupling Hamiltonian is $\hat{H}_{\rm int} = \lambda\hat{\sigma}_z\hat{a}^\dagger\hat{a}$, where the coupling strength λ sets the qubit-dependent cavity frequency shift. We will take the cavity to be at zero temperature, and ignore any intrinsic qubit dissipation, as we are interested only in the measurement backaction.

As discussed, making a weak σ_z measurement of the qubit involves strongly driving the cavity while monitoring the reflected light from the cavity via a homodyne measurement; the two possible values of σ_z will lead to two different average homodyne currents, which as time progresses can be resolved above the intrinsic noise in these currents [3,12]. We will focus exclusively on a weak nonlinearity $\Lambda \ll \kappa$ and a strong drive amplitude, such that the stationary average cavity photon number $\langle\hat{a}^\dagger\hat{a}\rangle \gg 1$ regardless of the initial qubit state. Apart from this constraint, we will not place any other restrictions on how small the qubit-cavity coupling λ must be.

Dephasing rate.—We first calculate the backaction dephasing of the qubit that occurs during such a measurement; this dephasing is a direct consequence of the intracavity photon-number fluctuations. If the qubit is initially in a σ_z eigenstate, it will remain in this state (due to the quantum nondemolition nature of the measurement); thus, from the cavity's perspective, the two qubit eigenstates simply correspond to a shift of the cavity resonance frequency by either $\pm\lambda$. In each case, the classically expected cavity amplitude α_σ ($\sigma = \uparrow, \downarrow$) will be given by [14,16]

$$\left[-\frac{\kappa}{2} + i(\Delta \mp \lambda + 2\Lambda|\alpha_{\uparrow/\downarrow}|^2)\right]\alpha_{\uparrow/\downarrow} = -if_0, \quad (2)$$

where f_0 is the amplitude of the cavity drive. Note that we focus on driving strengths small enough that we are below the bifurcation; i.e., there is only one classical solution α for a given Δ. By now, writing $\hat{a} = \hat{d} + \alpha_\sigma$ and using the fact that $|\alpha_\sigma| \gg 1$, we can approximate the cavity Hamiltonian corresponding to each qubit eigenstate by only keeping terms that are at most quadratic in $\hat{d}, \hat{d}^\dagger$. We thus obtain *two* linearized cavity Hamiltonians, corresponding to the two qubit states. Each has the general form of a degenerate parametric amplifier driven by an off-resonant pump [14,18,19]:

$$\hat{H}_\sigma = -\tilde{\Delta}_\sigma\hat{d}^\dagger\hat{d} + \frac{i}{2}(\tilde{g}_\sigma\hat{d}^\dagger\hat{d}^\dagger - \tilde{g}_\sigma^*\hat{d}\,\hat{d}), \quad (3)$$

where $\tilde{\Delta}_{\uparrow/\downarrow} = \Delta \mp \lambda + 4|\alpha_{\uparrow/\downarrow}|^2\Lambda$ is the effective pump detuning, and $\tilde{g}_\sigma = 2i\alpha_\sigma^2\Lambda$ is the parametric strength. As one approaches a point of bifurcation in the cavity (e.g., by increasing the drive strength f_0), the corresponding degenerate parametric amplifier Hamiltonian approaches the threshold of self-oscillation [14] (see inset of Fig. 2). For such operating points, incident waves on the cavity in the appropriate quadrature will be strongly amplified; this amplification is described by a photon number gain G which diverges as one approaches the bifurcation, as well as a narrow bandwidth $\kappa_{\rm slow} \sim \kappa/\sqrt{G}$ [14,17,19].

While the cavity evolution is easy to understand when the qubit is initially in a σ_z eigenstate, to calculate the dephasing rate we need to understand the cavity dynamics when the qubit is in a superposition of its eigenstates. We focus on the long-time qubit dephasing rate, which is defined as usual in terms of the decay of the qubit's off-diagonal density matrix elements in the long-time limit: $-|\ln \mathrm{Tr}(\hat{\rho}|\downarrow\rangle\langle\uparrow|)|/t \to \Gamma_\varphi$, where $\hat{\rho}$ is the density matrix describing the full system.

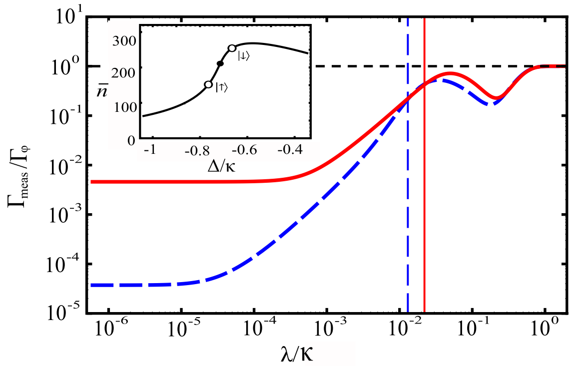

FIG. 2 (color online). Measurement efficiency ratio $\chi \equiv \Gamma_{\mathrm{meas}}/\Gamma_{\varphi}$ as a function of coupling strength for the same parameters as Fig. 1 ($G = 10^2$, red solid curve), and for an operating point closer to the bifurcation yielding $G \sim 10^3$ (blue dashed curve); the quantum limit is $\chi = 1$ (dashed black line). Vertical lines indicate the maximum λ for which the measurement time $1/\Gamma_{\mathrm{meas}}$ is longer than the detector response time $1/\kappa_{\mathrm{slow}} \sim \sqrt{G}/\kappa$. For $\lambda \to 0$, one misses the quantum limit by a large amount: $\chi \sim 1/G$. However, a small increase in coupling greatly improves this efficiency ratio. Inset: Average cavity photon number versus drive detuning Δ (parameters as in Fig. 1). The black point indicates the chosen working point. The two qubit states lead to two different effective values of Δ; these are shown as white circles for $\lambda = 0.05\kappa$.

To proceed, we first introduce $\hat{\rho}_{\uparrow\downarrow} = \mathrm{Tr}_{\mathrm{qb,bath}}(\hat{\rho}|\downarrow\rangle\langle\uparrow|)$, where the trace is over the qubit and cavity bath degrees of freedom. This is an operator acting in the cavity Hilbert space; its trace yields the off-diagonal element of the qubit density matrix, and hence can be used to obtain Γ_{φ}. We further transform $\hat{\rho}_{\uparrow\downarrow}$ by displacing away the two stationary classical cavity amplitudes α_{σ} associated with each qubit state. We thus obtain an operator $\hat{\tilde{\rho}}_{\uparrow\downarrow}$:

$$\hat{\tilde{\rho}}_{\uparrow\downarrow}(t) \equiv \hat{D}(-\alpha_{\uparrow})\hat{\rho}_{\uparrow\downarrow}(t)\hat{D}^{\dagger}(-\alpha_{\downarrow}), \tag{4}$$

where $\hat{D}(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \mathrm{H.c.})$ is the cavity displacement operator. One can show that in the long-time limit, the exponential decay of $\mathrm{Tr}\hat{\tilde{\rho}}_{\uparrow\downarrow}(t)$ also yields the dephasing rate Γ_{φ} [20].

It is now straightforward to rigorously derive the evolution equation of $\hat{\tilde{\rho}}_{\uparrow\downarrow}$, starting from the standard Linblad master equation describing the evolution of the cavity-plus-qubit density matrix [20] (see also Ref. [16]):

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\tilde{\rho}}_{\uparrow\downarrow} &= \kappa\mathcal{D}[\hat{d}]\hat{\tilde{\rho}}_{\uparrow\downarrow} - i(\hat{H}_{\uparrow}\hat{\tilde{\rho}}_{\uparrow\downarrow} - \hat{\tilde{\rho}}_{\uparrow\downarrow}\hat{H}_{\downarrow}) - \Gamma_{\varphi,0}\hat{\tilde{\rho}}_{\uparrow\downarrow} \\ &\quad + \kappa[(\alpha_{\uparrow} - \alpha_{\downarrow})\hat{\tilde{\rho}}_{\uparrow\downarrow}\hat{d}^{\dagger} - (\alpha_{\uparrow}^{*} - \alpha_{\downarrow}^{*})\hat{d}\hat{\tilde{\rho}}_{\uparrow\downarrow}].\end{aligned} \tag{5}$$

Here $\mathcal{D}[\hat{d}]\hat{\tilde{\rho}}_{\uparrow\downarrow} = \hat{d}\hat{\tilde{\rho}}_{\uparrow\downarrow}\hat{d}^{\dagger} - (\hat{d}^{\dagger}\hat{d}\hat{\tilde{\rho}}_{\uparrow\downarrow} + \hat{\tilde{\rho}}_{\uparrow\downarrow}\hat{d}^{\dagger}\hat{d})/2$ is the standard Lindblad superoperator describing cavity damping. The second and third terms in Eq. (5) correspond to the Hamiltonian evolution of the cavity in our doubly-displaced frame, where we have used $|\alpha_{\sigma}| \gg 1$ to linearize the two cavity Hamiltonians $\hat{H}_{\sigma}$ [cf. Eq. (3)]. The remaining terms on the rhs of Eq. (5) describe decoherence of the qubit resulting from the combination of the cavity drive and cavity dissipation, with $\Gamma_{\varphi,0} = (\kappa/2)|\alpha_{\uparrow} - \alpha_{\downarrow}|^2$.

For a linear cavity, the terms on the last line of Eq. (5) play no role, and the backaction dephasing rate is given completely by $\Gamma_{\varphi,0}$ (i.e., by the distinguishability of the two classical cavity amplitudes) [21]. For our case of a nonlinear cavity, the same is true *if* one neglects the parametric amplification terms in $\hat{H}_{\sigma}$ (proportional to $\hat{d}^2$ and $(\hat{d}^{\dagger})^2$), as then $\hat{\tilde{\rho}}_{\uparrow\downarrow}(t) = C\exp(-\Gamma_{\varphi,0}t)|0\rangle\langle 0|$ (where $|0\rangle$ is the vacuum state and C a constant) trivially solves Eq. (5). This is equivalent to finding that (for long times) the cavity state conditioned on the qubit is a coherent state $|\alpha_{\sigma}\rangle$. In this approximation, the backaction dephasing rate is given completely by the linear-cavity formula $\Gamma_{\varphi,0}$ [10,15,16]. However, such an approximation completely neglects the squeezing of noise by the nonlinear cavity. It is thus only valid for cavity parameters extremely far from any bifurcation, in regimes where the nonlinear cavity closely resembles a linear cavity.

Given the importance of noise squeezing, we go beyond the above approximation by retaining all terms in Eq. (5). Defining $\nu(t) = -\ln\mathrm{Tr}\hat{\tilde{\rho}}_{\uparrow\downarrow}(t)$, the long-time backaction dephasing rate will be given by $\Gamma_{\varphi} = \lim_{t\to\infty}\mathrm{Re}\,\nu(t)/t$. Setting $\delta\alpha = \alpha_{\uparrow} - \alpha_{\downarrow}$, the trace of Eq. (5) yields

$$\dot{\nu} = \Gamma_{\varphi,0} - i\langle\hat{H}_{\uparrow} - \hat{H}_{\downarrow}\rangle_{\uparrow\downarrow} + \kappa\langle\delta\alpha\cdot\hat{d}^{\dagger} - \mathrm{H.c.}\rangle_{\uparrow\downarrow}, \tag{6}$$

where we have defined the quasiexpectation value $\langle\hat{\mathcal{O}}\rangle_{\uparrow\downarrow} = \mathrm{Tr}(\hat{\mathcal{O}}\hat{\tilde{\rho}}_{\uparrow\downarrow})/\mathrm{Tr}(\hat{\tilde{\rho}}_{\uparrow\downarrow})$. As $\hat{\tilde{\rho}}_{\uparrow\downarrow}$ is not a true density matrix, the quasiexpectation of a Hermitian operator can be complex, and hence the second and third terms above can contribute to the backaction dephasing.

We now use the fact that Eq. (5) only involves terms that are at most quadratic in $\hat{d}, \hat{d}^{\dagger}$, and hence can be solved exactly by a $\hat{\tilde{\rho}}_{\uparrow\downarrow}$ which has a Gaussian form (i.e., its phase space representation is Gaussian [20]). Equation (5) thus reduces to a closed set of evolution equations for the quasimeans and covariances of $\hat{d}$, $\hat{d}^{\dagger}$ (see Ref. [20] for details). Solving these and substituting into Eq. (6) directly gives $\dot{\nu}$ and thus the dephasing rate. We stress that this approach is not perturbative in the coupling λ, and it does not neglect the noise squeezing expected near a cavity bifurcation. A similar procedure can be used to calculate the backaction dephasing of a linear cavity subject to both quantum and thermal noise [22].

To gain insight on the effect of increasing λ, we first use the above approach to calculate Γ_{φ} to order λ^4. For a cavity detuning Δ and drive f_0 chosen to be close to the bifurcation point, one finds

$$\Gamma_{\varphi} \simeq \frac{G}{\kappa}\left[\frac{2}{3}\lambda^2\bar{n} + \frac{\lambda^2 G^{3/2}}{9} - \frac{32}{27}\frac{\lambda^4\bar{n}G^3}{\kappa^2} - \frac{5}{81}\frac{\lambda^4 G^{9/2}}{\kappa^2}\right], \tag{7}$$

where $\bar{n} = |\alpha_0|^2$, α_0 is the zero-coupling classical cavity amplitude (i.e., solution to Eq. (2) at $\lambda = 0$), and $G \gg 1$ is the parametric photon-number gain (see Ref. [14]). At each order in λ, we have retained the leading terms in $\bar{n}$ and G. The first term here ($\propto \lambda^2 \bar{n}$) reproduces the results of Refs. [10,14,16] and arises solely from the linear-cavity dephasing rate $\Gamma_{\varphi,0}$ in Eq. (6). The second term in Eq. (7) (also order λ^2) is missed if one linearizes the qubit-cavity interaction, or makes the approximation $\Gamma_\varphi = \Gamma_{\varphi,0}$. More interesting are the λ^4 terms in Eq. (7). Surprisingly, these corrections are *negative*, suggesting the possibility of a relative suppression of dephasing with increased coupling (relative to the lowest-order-in-λ expression). The leading λ^4 corrections are completely due to the last line of Eq. (5), terms that would vanish if one ignored the squeezing of noise near the bifurcation. As such, the approximation $\Gamma_\varphi \simeq \Gamma_{\varphi,0}$ would predict both an incorrect sign and scaling with G of this term.

To see the full consequence of this behavior, we numerically solve Eq. (6) and the ordinary differential equations determining the needed averages; we use parameters corresponding to a weakly nonlinear cavity operated near a bifurcation point, similar to those realizable in experiment [10,17], and well within the regime of validity of our theory. In Fig. 1, one sees clearly that the backaction dephasing rate as a function of coupling (red) drops markedly below both the expectations from lowest-order perturbation theory (green), and below the linear-cavity formula $\Gamma_{\varphi,0}$ (grey). We have confirmed that this behavior is generic whenever one is close to a bifurcation in the cavity: *higher-order-in-* λ *terms yield a marked suppression of the dephasing rate.*

For further insight into the above behavior, we return to the physical picture that qubit dephasing is due to the photon-number fluctuations of the driven cavity. Treating these fluctuations classically and defining $m(t) \equiv \int_0^t dt' n(t')$ (where n is the cavity photon number), one finds that the off-diagonal qubit density matrix is directly proportional to the characteristic function of the probability distribution of m [23,24]. As such, the long-time qubit dephasing rate can be expressed in terms of the even cumulants of m, $\langle\langle m^{2j}\rangle\rangle$:

$$\Gamma_\varphi = \lim_{t\to\infty} \frac{1}{t} \sum_{j=1}^{\infty} (-1)^{j-1} \frac{(2\lambda)^{2j}}{(2j)!} \langle\langle m^{2j}\rangle\rangle. \tag{8}$$

This expansion also holds in the quantum case where $\hat{m}$ is an operator, if one now interprets the cumulants above using the standard Keldysh operator ordering [23,24].

Equation (8) implies that terms of order λ^4 and higher in Γ_φ are due to the non-Gaussian nature of intracavity photon number fluctuations. As shown in Eq. (7), the relative suppression of Γ_φ with λ is already presaged by the negative sign of the λ^4 term. From Eq. (8), we see that this implies a positive kurtosis, $\langle\langle m^4\rangle\rangle > 0$. Even a driven linear cavity in the classical limit has non-Gaussian intracavity photon number fluctuations and a positive kurtosis; this is because n is the square of a Gaussian random variable, the cavity amplitude [24]. A positive kurtosis signals a distribution which is more peaked than a Gaussian, and hence noise that generates less dephasing than Gaussian noise. In our nonlinear resonator, the non-Gaussian nature of the photon number fluctuations is strongly enhanced near bifurcation by the intrinsic nonlinearity of the system. $\langle\langle m^4\rangle\rangle$ remains positive (like a linear resonator), but is much larger than even that of a degenerate parametric amplifier near threshold [20].

Measurement rate and quantum limit.—For a measurement that occurs slowly on detector time scales, we can characterize the information gain of the measurement by a single measurement rate Γ_{meas}: how quickly do the distributions of the output homodyne current corresponding to each qubit eigenstate ($\uparrow$ or $\downarrow$) become distinguishable. Generalizing the standard weak-coupling expression [1–3] to a situation where the coupling is not perturbative but the measurement is still slow compared to internal detector timescales, we find (see Ref. [20] for details)

$$\Gamma_{\mathrm{meas}} = (\bar{I}_\uparrow - \bar{I}_\downarrow)^2/[4(S_{II,\uparrow} + S_{II,\downarrow})]. \tag{9}$$

Here $\bar{I}_\sigma$ is the average stationary homodyne current when the qubit is in the state $\sigma =\uparrow, \downarrow$, and similarly, $S_{II,\sigma}$ is the zero-frequency spectral density of homodyne current fluctuations when the qubit is frozen in the state σ. One can rigorously show that for arbitrary λ, the measurement efficiency ratio $\chi = \Gamma_{\mathrm{meas}}/\Gamma_\varphi \leq 1$ [20].

Using the linearized Hamiltonians $\hat{H}_\sigma$ given in Eq. (3) along with standard input-output theory [25] lets us evaluate Eq. (9) for an arbitrary value of the coupling; comparing against the dephasing rate then allows us to investigate the behavior of χ as a function of coupling. One finds that, similar to the linear-cavity dephasing rate $\Gamma_{\varphi,0}$, the measurement rate is largely determined by the classical amplitudes α_σ, and is hence a far weaker function of λ than the dephasing rate. The result is that there is a range of λ where higher-order terms significantly suppress the dephasing rate (over the perturbative expression), whereas the measurement rate is still determined by the leading-order expression [see Fig. 1(a)].

Shown in Fig. 2 is χ versus λ for the same parameters as in Fig. 1. In the limit of a vanishing coupling strength, one deviates strongly from the quantum limit [14]. However, the effective suppression of dephasing that occurs with a modest increase of coupling brings one within a factor of order unity of the ultimate quantum limit bound $\chi = 1$. We find that this behavior is generic for cavity operating points near bifurcation: increasing the coupling λ beyond the validity of leading-order perturbation theory allows one to make a weak measurement with a much higher efficiency than in the extreme weak coupling limit. On a physical level, this is a direct result of the non-Gaussian nature of photon number fluctuations in the driven cavity (namely, the large positive kurtosis).

Conclusions.—We have described a general method to calculate the measurement and dephasing rate of a qubit coupled to a nonlinear resonator that is not perturbative in the qubit-detector coupling and which accounts for cavity noise squeezing. By increasing the coupling to a regime where higher-order corrections are relevant, one can come significantly closer to the fundamental quantum limit on weak continuous qubit measurement.

This work was supported by NSERC and CIFAR.

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